A NUMERICAL ANALYSIS OF ERRORS IN A BOMBING SYSTEM

Ronald Chambless

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A Numerical Analysis of Errors
in a Bombing System

by

Ronald Chambless

March 1978

Thesis Advisor: Frank D. Faulkner

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Numerical Analysis of Errors in a Bombing System		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis: March 1978
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ronald Chambless		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey CA 93940		12. REPORT DATE March 1978
		13. NUMBER OF PAGES 53
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Naval Postgraduate School Monterey CA 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited. .

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An analysis based primarily on computations is conducted to determine errors in a bombing system similar to that currently under test and development by the Marine Corps.

There are many parameters in the system determining point of impact. Included are point of release and velocity at that point, air density, the mass and drag coefficient of the bomb, its cross sectional area, and the wind structure. Most values are not known with the desired accuracy but are estimated, say,

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
(Page 1) S/N 0102-014-6601 |

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

from radar measurements, for position.

This study estimates as much as possible the error in predicting the point of impact as a consequence of such factors as random uncorrelated noise in the radar measurements, errors in estimating density, errors in the wind structure, and errors due to initial conditions.

A NUMERICAL ANALYSIS OF ERRORS IN A BOMBING SYSTEM

by

Ronald Chambless
Captain, United States Marine Corps
B.S.E., Arkansas State University 1971

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE with major IN MATHEMATICS

from the

NAVAL POSTGRADUATE SCHOOL

March 1978

ABSTRACT

An analysis based primarily on computations is conducted to determine errors in a bombing system similar to that currently under test and development by the Marine Corps.

There are many parameters in the system determining point of impact. Included are point of release and velocity at that point, air density,the mass and drag coefficient of the bomb, its cross sectional area, and the wind structure. Most values are not known with the desired accuracy but must be estimated, say, from radar measurements, for position.

This study estimates as much as possible the error in predicting the point of impact as a consequence of such factors as random uncorrelated noise in the radar measurements, errors in estimating density, errors in the wind structure, and errors due to initial conditions.

TABLE OF CONTENTS

I. INTRODUCTION

This thesis developes and uses many of the mathematical techniques and relations that are needed to analyze the important errors in a bombing system. The AN/TPQ-27, described in Ref. 1 and Ref. 3, is the model for the analysis.

Precision bombing is a crucial means of supporting Marine Corps operations. It is a matter of life and death that it be as accurate and effective as possible. To ensure this the Marine Corps carries out extensive analysis and testing of its systems.

The complete analysis consists of two basic parts. First there is a mathematical modeling of the system; that is, equations must be formulated to describe its behavior and interreactions of its components. These must be based on the natural laws that govern the motions and on the accumulated experiences (some lore perhaps) with similar systems. The equations must be simple enough to allow the necessary solutions and computations, but they must also maintain the needed accuracy. This thesis takes up this part. Second, there must be extensive testing of the components and of the integrated system under conditions which simulate the field as closely as possible, but with extra measurements made to isolate and determine the sources of significant causes of error.

In this study the author has set up the basic mathematics for error analysis and carried out computations to show the typical effects of errors caused or observed at

various stages.

It is convenient to consider three classes of errors, depending on the time they are observed. Different analysis is required for each type.

First are the errors which are due to inaccuracies in the equations of motion during the period of fall. These may come from sources such as errors in estimating the drag coefficient, and variations in the air density and the wind structure from the assumed values.

Second, there may be significant errors in the position and velocity at the time of release. These may come from sources such as wind gusts near the time of release, faulty plane response, errors in the filtered estimates of position and velocity, and release or separation problems.

Third, there may be errors in the radar measurements and transients introduced in various ways. The errors may be inherent in the radar, they may be due to the changing aspect of the plane, to maneuvers, and/or to the filtering equations whose function is to reduce the effects of random errors. We will know only statistical properties of some of these, and for these we can only determine associated statistical properties of the associated miss distances.

A number of estimates were made in two or more ways, for comparison of the methods and to act as a check. For example the effects of variations in initial velocity were estimated in three ways: (1) by recalculating the trajectory, (2) by using variational equations, and (3) by using the adjoint or costate equations. For small changes the latter two are more accurate; for large changes, the former is better, but all are adequate over a considerable range. Estimates of errors in the filtered values of

position and velocity were also obtained in two basic ways: (1) by an analysis of the solutions to the difference equations that are used for filtering, and (2) by a numerical simulation of the solution, on the computer.

We need to make some remarks about notation and coordinate sets at this time. We will use an x-y-z Cartesian coordinate set fixed with respect to the earth. The program has a subroutine which corrects automatically for the earth's rotation so we can consider the set to be Newtonian. The z direction is the vertical. The plane is assumed to be flying horizontally, and its velocity (with respect to the ground) defines the y-axis.

Derivatives with respect to time are indicated by a dot over a variable, $\dot{x} = dx/dt$, etc. In the text, however, this is denoted by xdot, and a similar notation is used for other variables.

The ratio ρ/ρ_0 is used in the equations as the ratio of the air density at altitude z to the air density at ground level (We use an exponential value for this.). In the text these terms are written rho and rho(0) respectively. In the same way variations such as δx are written as DELx in the text.

It was not feasible to write subscripts, so that subscripted variables such as the drag coefficient, the x component of the wind velocity, the y component of the wind velocity, and others, were written Cd, Vwx, Vwy, etc., in the text; it is felt that this will not be confusing.

The notation was chosen to be in conformity with that in Ref. 1 and Ref. 3 as much as possible. Also, the terms impact and fall are both used interchangeably to denote conditions at the instant the bomb strikes the ground.

II. ERRORS DUE TO CHANGES IN DENSITY, WIND STRUCTURE, AND VALUE OF DRAG COEFFICIENT

When the ordnance is released it follows a path determined by natural laws. We approximate these as best we can to predict the point of impact. However, the equations of motion contain certain parameters and functions which may not be known, and the planned trajectory has a point of impact different from that calculated, and a miss results.

The effects of two of these parameters are investigated here. The first of these is the wind. Let us assume that we have a good estimate of the wind at the point of release (called the wind at altitude), because we know the velocity of the plane (from the radar), its airspeed, and its heading. However, the wind varies with altitude, and the structure of this function may be unknown under combat conditions. When testing, we can approximate this function by making extra meteorological measurements. Then by using various approximations to the wind, based on the wind at altitude, we can calculate the associated miss distance. An approximation that is commonly used is that the wind is constant or uniform, that constant being some fraction of the wind at altitude. The lore indicates that eight-tenths is a good fraction; higher values, approaching one, have been suggested [Ref. 4]. Under the conditions of our example eight-tenths seems like a good choice. However, the component of the wind normal to the wind at altitude often causes a large error that there is no way to correct knowing just the wind at altitude. Only a further study of wind structures will show when bombing is ineffectual without more detailed knowledge of the wind.

A second factor whose effects we can estimate readily is the drag parameter. There is a single drag parameter, which involves the air density at some reference level, the cross sectional area of the bomb and its mass, and a drag coefficient, Cd, depending primarily on its shape. We can estimate the effects of errors in this parameter directly.

There are many other sources of error for which no simple analysis exists; some of these are discussed later in this chapter.

A. EQUATIONS OF MOTION FOR A PROJECTILE

Reference 1 describes in detail the equations involved in the ballistic integration. These are the classical equations of motion for a non-lifting projectile:

$$\ddot{x} = -A/m \, \rho \, C_d \, V \, (\dot{x} - V_{wx}) \qquad (1a)$$

$$\ddot{y} = -A/m \, \rho \, C_d \, V \, (\dot{y} - V_{wy}) \qquad (1b)$$

$$\ddot{z} = -g - A/m \, \rho \, C_d \, V \, \dot{z} \qquad (1c)$$

where A is the cross sectional area of the projectile, m is the mass of the projectile, g is the acceleration due to gravity; rho, Cd, xdot, ydot, zdot, Vwx, and Vwy have been previously defined; and V, the airspeed, is:

$$V = \sqrt{(\dot{x} - V_{wx})^2 + (\dot{y} - V_{wy})^2 + \dot{z}^2} \qquad (2)$$

Equations (1a), (1b), and (1c) were integrated numerically to obtain final x, y, and z coordinates and

hence, to estimate the bomb impact point.

B. BASIC ASSUMPTIONS AND CONDITIONS

It was decided to use values for various parameters that were selected arbitrarily in a reasonable range, rather than treat a specific missle, whose details would be classified.

1. Altitude and Airspeed

In all cases the aircraft was assumed to be flying straight and level at an altitude of 20,000 feet with an airspeed of 350 feet per second. In those cases where a wind structure was considered the initial velocity (with respect to the earth) was altered to take into account the wind velocity.

2. Drag Coefficient and Constant Values

The drag coefficient, Cd; the cross sectional area of the projectile, A; the mass of the projectile, m; and the air density at ground level, rho(0) ; were assumed for the majority of the work to take on values such that the important ratio C=(A/m)(Cd)(rho(0)) had the value 0.000025. This figure was considered to be a good approximation to some typical ordnance types without requiring the thesis to be classified.

3. Air Density

It was assumed the decrease in air density with

altitude was given by the factor exp(-z ln2/18000). This function was compared with the ratio given in Ref. 1, and it was felt that the exponential expression was satisfactory (and simplified programming).

4. Wind Structure

Reference 4 details wind structure data obtained by Camp Pendleton Marine Corps Base in 1975. The data was collected twice daily on 25 March, 28 March, and 12 April. Wind velocity in knots and wind angle were determined at 2,000 foot intervals from ground level to 36,000 feet. All reference within this work to a "variable wind structure" applies to the data in Ref. 4 collected 25 March, 1975, at 11:00 unless otherwise noted.

C. COMPARISON OF SEVERAL WIND CONDITIONS

Several trajectories were computed based on two actual wind structures. In each case directions were chosen for the plane corresponding to a headwind, a crosswind, and a tailwind. In each case trajectories were computed with a uniform wind equal to the wind at altitude, and one equal to eight-tenths wind at altitude. In two cases various other uniform winds were also assumed.

In all calculated trajectories the following equations and terms were used:

$$\mathring{x} = u \qquad (3a)$$

$$\mathring{y} = v \qquad (3b)$$

All trajectories were calculated using a modified Euler Method [Ref.2] with a time step, DT=1.0.

Since the wind velocity components in Ref. 4 are provided at 2,000 feet intervals only, a linear interpolation scheme was devised to compute wind velocity components as the bomb fell.

Three trajectories using the wind structure at 11:00, 25 March, 1975, were then computed for three conditions: a plane flying into a headwind, a plane flying with a crosswind, and a plane flying with a tailwind. Next, wind at altitude was considered as the constant wind velocity throughout as opposed to using a variable wind structure, and the three trajectories computed again. Finally, the three trajectories were computed again assuming the wind to be uniformly eight-tenths that at altitude.

The trajectories based on the variable wind structure were used as trajectories of reference and the errors due to the approximations calculated. Corresponding trajectories using the stratified wind data of 25 March, 1975, 14:00, contained in Ref. 4 were then calculated under the conditions described above, for a comparison.

If the wind at altitude is known, and is all that is known about the wind, the only uniform wind we may reasonably choose is one which is some fraction of the wind at altitude. It was felt that the change in impact point due to changing this fraction would be approximately linear. To check this two trajectories were calculated using nine-tenths and zero respectively, as fractions. The resulting trajectory using nine-tenths led to a point of impact which was approximately the midpoint between the points due to fractions of eight-tenths and one, indicating

the change was approximately linear. A typical plot (Figure 1) showed that in the example considered, the value of eight-tenths was near the optimum. It is felt no other value would result in a significantly smaller error in any of the cases considered.

While Ref. 4 points out that fractions of wind at altitude closer to unity are better for certain bomb loads, the results indicate that for these wind structures, eight-tenths is near the best choice for use in the absence of stratified wind data. The resulting error was near the minimum in all cases for this choice.

For these wind structures there was always a sizeable error when a constant wind was assumed, due to changes in the direction of the wind with altitude. More analysis with different wind observations is needed to establish the validity of such a universal factor and the general errors due to the varying direction of the wind.

Some additional results of the analysis are contained in TABLES I and II.

In the next chapter another method is given for estimating the effects of the changes discussed above, using methods like those developed by G. A. Bliss for corrections in artillery.

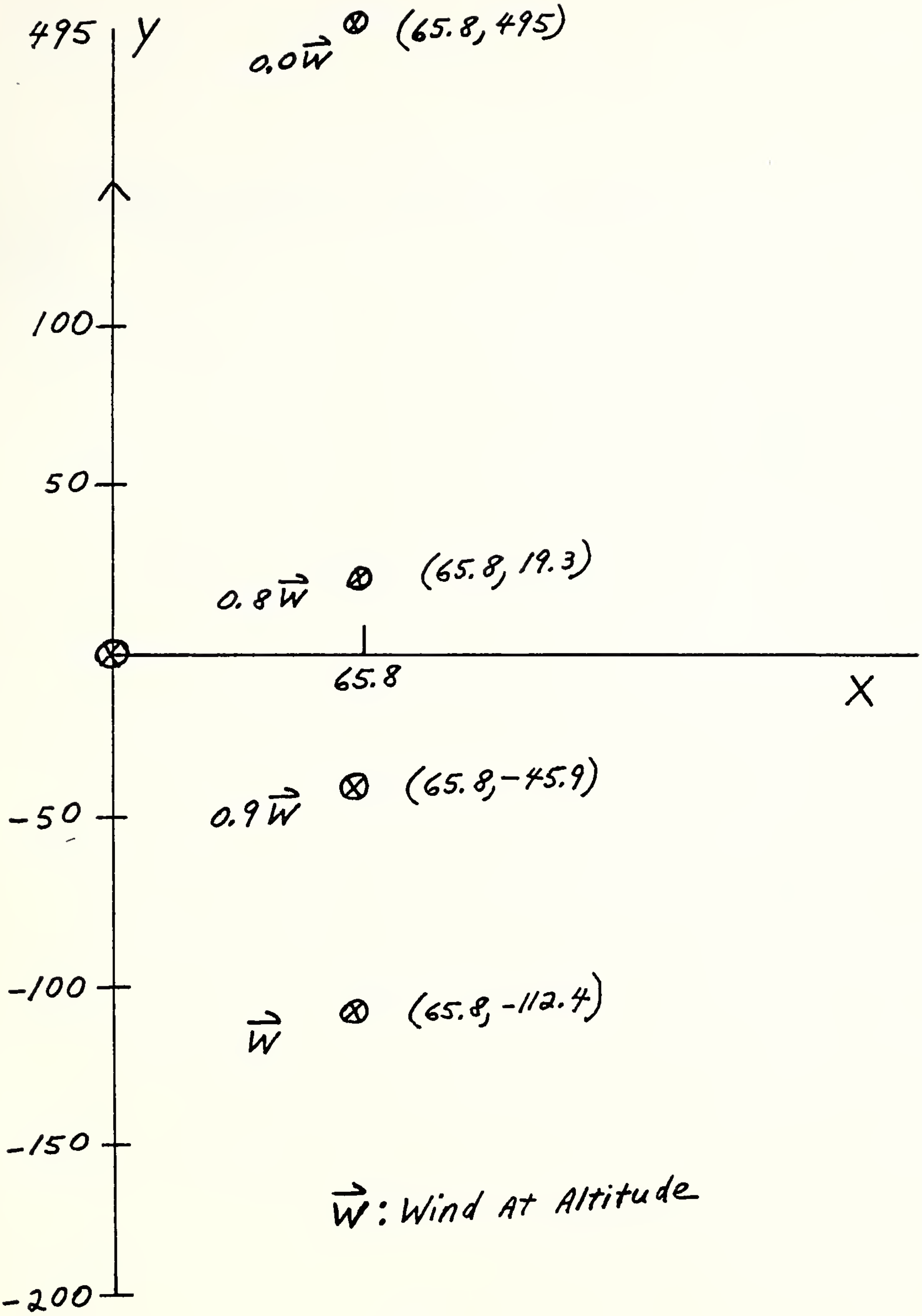

Figure 1

A typical Plot of the Errors From the Variable Trajectory Impact Point (in Ft.) When Various Fractions of Wind at Altitude Are Assumed

STRATIFIED WIND DATA FROM 25 MARCH, 75, 11:00

type wind	wind vel used	x	y	x error	y error
tail	variable	67.754	15519.350	N/A	N/A
tail	wind at alt	-.028	15629.180	-67.8	110
tail	.8 wind at alt	-.023	15499.350	-67.8	-20
cross	variable	-409.653	10994.860	N/A	N/A
cross	wind at alt	-518.572	10922.150	-108.9	-72.1
cross	.8 wind at alt	-412.139	10929.470	-2.5	-65.4
head	variable	-65.814	7488.727	N/A	N/A
head	wind at alt	0.0	7376.344	65.8	-112.4
head	.8 wind at alt	0.0	7507.988	65.8	19.3

Table I

X and Y Errors (in Ft.) for Wind at Altitude and .8 Wind at Altitude Trajectories Versus Variable Trajectory Under Various Wind Conditions

STRATIFIED WIND DATA FROM 25 MARCH, 75, 14:00

type wind	wind vel used	x	y	x error	y error
tail	variable	57.813	15865.250	N/A	N/A
tail	wind at alt	-.028	16010.070	-57.8	144.8
tail	.8 wind at alt	-.023	15868.360	-57.8	3.11
cross	variable	-422.153	10874.430	N/A	N/A
cross	wind at alt	-566.438	10806.460	-144.3	-68.0
cross	.8 wind at alt	-449.604	10815.100	-27.5	-59.0
head	variable	-56.073	7142.984	N/A	N/A
head	wind at alt	0.0	6995.461	56.0	-148.0
head	.8 wind at alt	0.0	7139.324	56.0	-3.7

Table II

X and Y Errors (in Ft.) for Wind at Altitude and .8 Wind at Altitude Trajectories Versus Variable Trajectory Under Various Wind Conditions

D. COMPARISON OF VALUES OF THE CONSTANT C

The value C=(A/m)(Cd)(rho(0))=0.000025 was, as stated earlier, used for the majority of the computation. In this section this parameter was varied to determine the effects.

Trajectories were computed with various values of C, namely 0, .0000125, .000025, and .000050, to determine the associated change in point of impact.

As indicated in TABLE III, the function is nearly linear over a wide range, and hence changes in point of impact could be estimated by linear approximations.

C	Y	CHANGE IN IMPACT POINT
.000025	11502.730	N/A
.000050	10707.760	794.97
.0000125	11928.490	425.76
0.0	12373.230	870.5

Table III

Change in Point of Impact (in Ft.) as Result of Varying the Constant C (Trajectory with C = 0.000025 is Nominal Trajectory)

III. EFFECTS OF INITIAL CONDITIONS ON POINT OF FALL

From the point of view of control the time of release of the bomb is the crucial instant. Conditions at this time determine success or failure, and nothing further can be done to improve bombing accuracy.

In this chapter a study is made of the changes, or errors in the point of fall, as a consequence of changes or errors in initial conditions. By initial conditions are meant the conditions at the instant of release from the plane.

The effects of changes in the initial values of x and y on the point of impact are obvious. The effects of changes in u, v, w, and z are not so obvious; these effects are estimated in three ways and the results compared. The first way is to take a typical trajectory and then recompute the trajectory several times, each time changing one initial condition by a "small amount". The second way is to use the variational equations. These were integrated, using the appropriate initial conditions in order to derive the differential estimates of the changes in the point of impact. The third way is to define the adjoint or costate equations. These are integrated backwards, with appropriate terminal conditions, to define the so-called influence coefficients, which may be interpreted as the partial derivatives of the final conditions with respect to the initial conditions.

Comparisons of the results were carried out with three reference or nominal trajectories. In one trajectory there

was assumed to be no wind; in the second the wind was assumed to be uniformly eight-tenths that at altitude; in the third the wind function was assumed to be that observed at Camp Pendleton on 25 March, 1975, at 11:00.

In all cases all of the methods gave very similar results.

A. VARIATIONAL EQUATIONS

The variational equations are derived as follows. Consider the equations of motion:

$$\mathring{u} = -C\rho V(u - V_{wx}) \tag{1a}$$

$$\mathring{v} = -C\rho V(v - V_{wy}) \tag{1b}$$

$$\mathring{w} = -C\rho V w - g \tag{1c}$$

where u, v, w, C, rho, V,Vwx, and Vwy have been previously defined. Then let us consider the first-order effects if we change the variables by small amounts in some way, but require that the new variables satisfy the equations of motion. The first-order terms in the change, DELx, etc., must then satisfy the equations:

$$\delta\mathring{x} = \delta u \tag{2a}$$

$$\delta\mathring{y} = \delta v \tag{2b}$$

$$\delta\mathring{z} = \delta w \tag{2c}$$

$$\delta \mathring{u} = -C\rho' \delta z\, V(u - V_{wx}) - C\rho\,(u - V_{wx}) \qquad (2d)$$

$$\left\{ \frac{(u - V_{wx})(\delta u - \delta V_{wx}) + (v - V_{wy})(\delta v - \delta V_{wy}) + w\,\delta w}{V} \right\}$$

$$- C\rho V(\delta u - \delta V_{wx})$$

$$\delta \mathring{v} = -C\rho' \delta z\, V(v - V_{wy}) - C\rho\,(v - V_{wy}) \qquad (2e)$$

$$\left\{ \frac{(u - V_{wx})(\delta u - \delta V_{wx}) + (v - V_{wy})(\delta v - \delta V_{wy}) + w\,\delta w}{V} \right\}$$

$$- C\rho V(\delta v - \delta V_{wy})$$

$$\delta \mathring{w} = -C V w \rho' \delta z - C\rho\, w \qquad (2f)$$

$$\left\{ \frac{(u - V_{wx})(\delta u - \delta V_{wx}) + (v - V_{wy})(\delta v - \delta V_{wy}) + w\,\delta w}{V} \right\}$$

$$- C\rho V \delta w \, .$$

These are called the variational equations. It may be noted that they are linear but with variable coefficients whose values must be obtained from the original trajectory. If the variations in the wind are zero, the equations are also homogeneous.

1. Effects of Changes in v

As a first example let us consider the effects of changes in the initial speed, v(0). Let us take as a standard or nominal path one with no wind. The values of z, u, v, w when they occur in the variational equations are from this path.

Let us consider DELv(0)=1, and all other variations to be zero initially. We will integrate to the terminal time tf, the time of fall on the original path. We find that DELy(tf)=31.30. However, DELz(tf)=1.34, and hence the differential change in terminal time (denoted Dtf) is Dtf=-DELz(tf)/w(tf)=0.001. The differential change in y (denoted Dyf) then requires a secondary term due to the change in terminal time: Dyf=DELy(tf)+v(tf)(Dtf)=31.63. We see that the change in terminal time has a secondary effect.

The variation in x in this case is zero.

2. Effects of Other Changes

In a similar manner the effects of changes in other initial conditions were studied. The variations DELu(0), DELz(0), and DELw(0) were each separately set equal to one initially, while all of the other variations were set to zero initially; we integrated to terminal time, tf, and

computed Dxf and Dyf in each case. We then computed the error in impact point due to the differential changes in x and y.

Identical calculations were performed considering the three separate nominal trajectories, and as indicated in TABLE IV, differences in figures among the three trajectories are quite minimal.

NOMINAL TRAJ.	ERROR WHEN DELV(0)= 1.0	ERROR WHEN V(0) INCREASED 10.0	ERROR WHEN DELZ(0)= 1.0	ERROR WHEN Z(0) INCREASED 10.0
no wind	31.633	315.938	.283	2.85
.8 wind at alt	31.767	317.0	.246	2.495
variable wind	31.671	316.31	.245	3.845

	ERROR WHEN DELW(0)= 1.0	ERROR WHEN W(0) INCREASED 10.0	ERROR WHEN DELU(0)= 1.0	ERROR WHEN U(0) INCREASED 10.0
no wind	9.535	96.305	32.867	328.646
.8 wind at alt	8.372	84.532	32.799	328.0
variable wind	8.306	83.60	32.728	327.4

Table IV

Comparison of Errors in Impact Point (in Ft.) When Errors in Various Initial Conditions are Assumed

B. THE ADJOINT SYSTEM

The adjoint system for the equations of motion was derived, and the influence coefficients analyzed as a means of cross checking the results of the variational equation analysis. The nominal trajectory for this comparison had no wind.

1. Derivation of Influence Coefficients

Let us introduce six new unspecified variables P1,P2,P3,P4,P5,and P6 to be defined later. The hamiltonian for the system is defined:

$$H = P_1 u + P_2 v + P_3 w - \rho C V \qquad (3)$$

$$\left[P_4 \left(u - V_{wx} \right) + P_5 \left(v - V_{wy} \right) + P_6 w \right] - P_6 g$$

where u, v, w, rho, C, Vwx, and Vwy have been previously defined. Now, let us always choose the P's to be solutions to the differential equations:

$$\mathring{P}_i = - \frac{\partial H}{\partial x_i} \qquad (4)$$

where x1=x, x2=y, x3=z, x4=u, x5=v, and x6=w.

Then

$$\mathring{P}_1 = \frac{-\partial H}{\partial x} = 0 \qquad (5a)$$

$$\overset{\circ}{P}_2 = \frac{-\partial H}{\partial y} = 0 \tag{5b}$$

$$\overset{\circ}{P}_3 = -\frac{\partial H}{\partial z} = -\left[-\frac{\partial \rho}{\partial z} CV\left(P_4\left(u - V_{wx}\right) + P_5\left(v - V_{wy}\right) + P_6 w\right)\right] \tag{5c}$$

$$\overset{\circ}{P}_3 = \frac{\partial \rho}{\partial z} CV\left[P_4\left(u - V_{wx}\right) + P_5\left(v - V_{wy}\right) + P_6 w\right] \tag{5d}$$

$$\overset{\circ}{P}_4 = -\frac{\partial H}{\partial u} = -P_1 + \rho C\left\{VP_4 + \left[P_4\left(u - V_{wx}\right) + P_5\left(v - V_{wy}\right) + P_6 w\right]\left(\frac{u - V_{wx}}{V}\right)\right\} \tag{5e}$$

$$\overset{\circ}{P}_4 = -P_1 + \rho CVP_4 + \rho C\left(\frac{u - V_{wx}}{V}\right)\left[P_4\left(u - V_{wx}\right) + P_5\left(v - V_{wy}\right) + P_6 w\right] \tag{5f}$$

$$\overset{\circ}{P}_5 = \frac{-\partial H}{\partial v} = -\left\{P_2 - \rho C\left\{VP_5 + \left[P_4\left(u - V_{wx}\right) + P_5\left(v - V_{wy}\right) + P_6 w\right]\left(\frac{v - V_{wy}}{V}\right)\right\}\right\} \tag{5g}$$

$$\overset{\circ}{P}_5 = -P_2 + \rho CVP_5 + \rho C\left(\frac{v - V_{wy}}{V}\right)\left[P_4\left(u - V_{wx}\right) + P_5\left(v - V_{wy}\right) + P_6 w\right] \tag{5h}$$

$$\overset{\circ}{P}_6 = \frac{-\partial H}{\partial w} = -\left\{P_3 - \rho C\left\{VP_6 + \left[P_4\left(u - V_{wx}\right) + P_5\left(v - V_{wy}\right) + P_6 w\right]\frac{w}{V}\right\}\right\} \tag{5i}$$

$$\overset{\circ}{P}_6 = -P_3 + \rho CVP_6 + \frac{\rho Cw}{V}\left[P_4\left(u - V_{wx}\right) + P_5\left(v - V_{wy}\right) + P_6 w\right]. \tag{5j}$$

This does not define a solution yet. If we define values for all six components at some particular time, a particular solution will be determined completely, though we may have to integrate numerically to obtain it.

The adjoint variables and the variations satisfy an important identity

$$\frac{d}{dt} \sum_{i=1}^{6} p_i \, \delta x_i = 0 \tag{6}$$

which is easily verified, whenever the DELxi are solutions to the homogeneous variational equations.

This relation is the basis for obtaining what are sometimes called influence coefficients or sensitivity factors.

To illustrate how these are obtained, let us derive the sensitivity factors for the final value of y, or x2. First, let us observe that if we integrate equation (6) from zero to tf we get

$$\left(\sum_{i=1}^{6} p_i \, \delta x_i \right)_{t=t_f} = \left(\sum_{i=1}^{6} p_i \, \delta x_i \right)_{t=0} \tag{7}$$

Now, let us choose a solution to the adjoint such that at time tf the coefficient of DELy=DELx2 is P2(tf)=1, but all other Pi(tf) are zero. To generate this solution we must integrate backwards to t=0. For this particular solution to the adjoint system we get then from equation (7)

$$\delta y(t_f) = \left(\sum_{i=1}^{6} p_i \, \delta x_i \right)_{t=0} \tag{8}$$

That is, for this particular solution the components Pi(0) yield the influence of DELxi(0) on DELy(tf), and hence the name.

Several comments may be made. There is one such solution vector Pi for each variable x, y, z, u, v, and w. These were calculated for their corresponding variables. The values obtained in this way for the final variations should be the same as those obtained from the variational equations except for errors in numerical integration due to roundoff, truncation, etc.

2. Analysis of Results

Now, the influence coefficients are actually the various partial derivatives of the argument at final time with respect to the arguments x, y, z, u, v, and w at initial time. For instance, the influence coefficient associated with x defines a six component vector whose components are the partial of x at final time with respect to x at initial time, the partial of x at final time with respect to y at initial time, the partial of x at final time with respect to z at initial time, the partial of x at final time with respect to u at initial time, the partial of x at final time with respect to v at initial time, and the partial of x at final time with respect to w at initial time. The values of all these partial derivatives or influence coefficients are given in TABLE V.

These coefficients serve as a cross check for the analysis of the variational equations. For instance, the variational equations with DELu(0)=1.0 yielded a value of DELx at tf of 32.867 while the adjoint indicates that the partial of x at final time with respect to u at initial time is 32.870, which agree to four figures. Likewise, the

variational equations with DELv(0)=1.0 yielded a DELy at tf of 31.297 while the adjoint indicates that the partial of y at final time with respect to v at initial time is 31.298. Other values were checked similarly.

In conclusion, we found that the three methods of calculating the effects of initial conditions on the point of fall: (1) recomputing the trajectory, (2) using the variational equations, and (3) using the adjoint are satisfactory within the range of interest. We did find that the differential method augmented by the correction for the change in terminal time of fall is slightly more accurate, but the increase is really not significant. Finally, we see that in a computer age such as today the effects can just as easily be calculated by changing the initial condition and programming the trajectory again, although the use of the variational equations is still an important tool for analyzing such problems.

$\partial x_f/\partial(\ )\big\vert_{x_0,y_0,z_0,u_0,v_0,w_0}$	$\partial y_f/\partial(\ )\big\vert_{x_0,y_0,z_0,u_0,v_0,w_0}$
1.0	0.0
0.0	1.0
0.0	.052
32.870	0.0
0.0	31.298
0.0	2.255

$\partial z_f/\partial(\ )\big\vert_{x_0,y_0,z_0,u_0,v_0,w_0}$	$\partial u_f/\partial(\ )\big\vert_{x_0,y_0,z_0,u_0,v_0,w_0}$
0.0	0.0
0.0	0.0
.920	0.0
0.0	.667
1.338	0.0
28.983	0.0

$\partial v_f/\partial(\ )\big\vert_{x_0,y_0,z_0,u_0,v_0,w_0}$	$\partial w_f/\partial(\ )\big\vert_{x_0,y_0,z_0,u_0,v_0,w_0}$
0.0	0.0
0.0	0.0
0.003	.007
0.0	0.0
.616	.064
.147	.301

Table V

Adjoint Sensitivity Factors

C. COMMENTS

The problems of Chapter II can also be solved by the methods of this chapter. Indeed, Bliss developed the techniques associated with the adjoint to handle just such problems, while he was at Aberdeen Proving Grounds during World War I [Ref. 7].

Let us consider the effects of varying the wind profile from its values on some nominal path. The variational equations, (2a) through (2f), apply directly. The values of z, u, Vwx, etc., are those from the given nominal path, and the values of DELVwx, DELVwy are the values on the new path minus those on the nominal path.

If the adjoint equations are used, the equation for, say DELy(tf), reduces to

$$\delta y(t_f) = \int_0^{t_f} (C/V) \exp(-z \ln 2/18000) \left[P_4 (u - V_{wx}) \delta V_{wx} + P_5 (v - V_{wy}) \delta V_{wy} \right] dt \tag{9}$$

where the solutions to the adjoint are those associated with y, for which P2(tf)=1, and the final value of the other Pi are zero. The evaluation of the integral is simplier than integrating a system of differential equations.

IV. ANALYSIS OF FACTORS INFLUENCING AIRCRAFT POSITION AND VELOCITY ESTIMATES

The effects of some errors in radar measurements of position on the filtered values for position and velocity are investigated here. Also included in the study are errors due to some simple maneuvers and associated transients. Reference 1 provides the equations used in the AN/TPQ-27 system to predict and filter the aircraft position and velocity. The analysis is generally carried out for a single coordinate, y; by definition xdot is zero and ordinarily zdot will also be zero.

A. RANDOM UNCORRELATED NOISE

The first error analyzed is that due to random uncorrelated errors or noise in the radar measurements. The variance of the resulting errors was determined in terms of the variance of the noise, in two ways. The first was purely computational. A set of random numbers, v(k), with mean value zero, variance one, and gaussian distribution was generated. These were used like measurements or observations in the filtering equations. Sequences of filtered estimates for y and ydot were generated and their means (which were near zero) and variances were calculated on the computer. The second method was to analyze the filtering equations to estimate the variances of the filtered estimates of y and ydot.

1. A Numerical Approach

A routine for generating random numbers was called from the computer library. It generated random numbers v(k), k=1,2,....etc. These values were used as a forcing function, y(k), in the filtering equations. The position and velocity, y and ydot, were initially set to zero, and then their filtered values calculated iteratively. Numerical estimates were then made for the standard deviations of the filtered values of y and ydot using those calculated values as the sample [Ref. 5]. It was then found with this strictly numerical analysis that the standard deviation of the sample of filtered values of y was approximately .245 σ_v and the standard deviation of the sample of filtered values of ydot was approximately .118 σ_v .

2. An Analytic Approach

The prediction equations are:

$$\hat{y}_{m,m-1} = \hat{y}_{m-1,m-1} + \hat{\dot{y}}_{m-1,m-1}\,\Delta t \qquad (1)$$

$$\hat{\dot{y}}_{m,m-1} = \hat{\dot{y}}_{m-1,m-1} \qquad (2)$$

The filtering equations are:

$$\hat{y}_{m,m} = \hat{y}_{m,m-1} + \alpha\left(y_m - \hat{y}_{m,m-1}\right) \qquad (3)$$

$$\hat{\dot{y}}_{m,m} = \hat{\dot{y}}_{m,m-1} + \beta/\Delta t\left(y_m - \hat{y}_{m,m-1}\right) \qquad (4)$$

where $\hat{y}_{m,m}$ is the estimate of y(tn) made at time tm; the corresponding estimate for ydot is $\hat{\dot{y}}_{m,m}$; alpha=.1, beta=.0052, and DT=.125 [Ref. 1]. Now, the substitution of the right side of (1) into (3), and the substitution of the

right sides of (1) and (2) into (4) yields:

$$\hat{y}_{m,m} = \hat{y}_{m-1,m-1} + \hat{\dot{y}}_{m-1,m-1}\,\Delta t + \alpha\left(y_m - \hat{y}_{m-1,m-1} - \hat{\dot{y}}_{m-1,m-1}\,\Delta t\right) \tag{5}$$

$$= (1-\alpha)\,\hat{y}_{m-1,m-1} + \Delta t(1-\alpha)\,\hat{\dot{y}}_{m-1,m-1} + \alpha\, y_m$$

$$\hat{\dot{y}}_{m,m} = \hat{\dot{y}}_{m-1,m-1} + \beta/\Delta t\left(y_m - \hat{y}_{m-1,m-1} - \hat{\dot{y}}_{m-1,m-1}\,\Delta t\right) \tag{6}$$

$$= -\beta/\Delta t\ \hat{y}_{m-1,m-1} + (1-\beta)\,\hat{\dot{y}}_{m-1,m-1} + \beta/\Delta t\ y_m .$$

These are the equations for the filtered estimates of y and ydot, in terms of the observations.

Next, the following substitutions were made:

$$\hat{y}_{m,m} = \eta_m \tag{7}$$

$$\hat{\dot{y}}_{m,m} = \frac{\zeta_m}{\Delta t} \tag{8}$$

$$\zeta_m = \hat{\dot{y}}_{m,m}\,\Delta t\ , \tag{9}$$

then (5) and (6) become

$$\eta_m = (1-\alpha)\,\eta_{m-1} + (1-\alpha)\,\zeta_{m-1} + \alpha\, y_m \tag{10}$$

$$\zeta_m = -\beta\,\eta_{m-1} + (1-\beta)\,\zeta_{m-1} + \beta\, y_m . \tag{11}$$

One of the variables, say, zeta may be eliminated as follows. Shift the indices in (10) and (11) by one, to get

$$\eta_{m-1} = (1-\alpha)\eta_{m-2} + (1-\alpha)\zeta_{m-2} + \alpha V_{m-1} \tag{12}$$

$$\zeta_{m-1} = -\beta\eta_{m-2} + (1-\beta)\zeta_{m-2} + \beta V_{m-1} \quad . \tag{13}$$

When zeta is eliminated among these equations the equation for eta becomes

$$\eta_m = (2-\alpha-\beta)\eta_{m-1} - (1-\alpha)\eta_{m-2} + \alpha V_m + (\beta-\alpha)V_{m-1} \quad . \tag{14}$$

If we set

$$A = 2-\alpha-\beta \tag{15}$$

$$B = -(1-\alpha) \tag{16}$$

$$C = \beta - \alpha \tag{17}$$

this becomes

$$\eta_m = A\eta_{m-1} + B\eta_{m-2} + \alpha V_m + CV_{m-1} \quad . \tag{18}$$

In the same way the equation for zeta may be obtained,

$$\zeta_m = A\zeta_{m-1} + B\zeta_{m-2} + \beta V_m + DV_{m-1} \tag{19}$$

where

$$D = -\beta \quad . \tag{20}$$

In order to estimate the variance of eta we need to express eta in terms of the uncorrelated observations. To do this let us shift indices in equation (18);

$$\eta_{m-1} = A\eta_{m-2} + B\eta_{m-3} + \alpha V_{m-1} + CV_{m-2} \tag{21}$$

and eliminate η_{m-1} from (18) with this;

$$\eta_m = (A^2+B)\eta_{m-2} + AB\eta_{m-3} + \alpha V_m + (A\alpha + C)V_{m-1} + ACV_{m-2} \quad . \tag{22}$$

Repeated substitutions of this type lead to a series which may be written

$$\eta_m = \alpha V_m + (A\alpha + C)V_{m-1} + (A_2\alpha + AC)V_{m-2} + (A_3\alpha + A_2C)V_{m-3} + \ldots\ldots \tag{23}$$

or

$$\eta_m = E_0V_m + E_1V_{m-1} + E_2V_{m-2} + \ldots\ldots E_mV_{m-m} + \ldots\ldots \quad . \tag{24}$$

The coefficients A, B, and E are defined by the recursion formulas and initial values indicated below.

$$A_m = AA_{m-1} + B_{m-1} \tag{25}$$

$$B_m = BA_{m-1} \tag{26}$$

$$E_m = A_m \alpha + A_{m-1} C \tag{27}$$

$$A_0 = 1 \tag{28}$$

$$B_0 = 0 \tag{29}$$

$$E_0 = \alpha \tag{30}$$

If we square both sides and consider the expected values we get

$$\sigma_\eta^2 = \left(\sum_0^\infty E_m^2 \right) \sigma_\nu^2 \quad . \tag{31}$$

A similar operation for zeta leads to a recursion formula

$$F_m = A_m \beta + A_{m-1} D \tag{32}$$

where the A's and B's have been previously defined and

$$F_0 = \beta \quad . \tag{33}$$

We then get

$$\sigma_\zeta^2 = \left(\sum_0^\infty F_m^2 \right) \sigma_\nu^2 \quad . \tag{34}$$

3. Comparison of Results Obtained

The series represented in equations (31) and (34) were summed iteratively. Calculated standard deviations for the sample of filtered values of y and of ydot were found to be .275 σ_v and .095 σ_v respectively. The two methods of analysis yielded values differing by approximately eleven (11) per cent for the standard deviation of y and by approximately twenty (20) per cent for the standard deviation of ydot. The difference of twenty (20) per cent is somewhat disappointing.

B. PERIODIC ERRORS

Measurements are also often subject to some sort of periodic error, or noise. This was approximated by a sin wave and the effects on the filtered values of y and ydot estimated. Various frequencies were considered and the resulting error in position and velocity was determined for each.

1. The Analysis

The filtering equations (10) and (11) were of the form:

$$\eta_m = a\eta_{m-1} + a\zeta_{m-1} + \alpha V_m \qquad (35)$$

$$\zeta_m = -\beta\eta_{m-1} + c\zeta_{m-1} + \beta V_m \qquad (36)$$

where

$$a = 1-\alpha \tag{37}$$

$$c = 1-\beta \quad . \tag{38}$$

The errors in measurements were expressed in complex form

$$V_m = e^{m\theta i} \tag{39}$$

where theta is the frequency. Multiple values of pi, differing by factors of two, were used for theta, and the amplitudes of the resulting steady state solution were calculated.

The solutions were assumed to have the same form,

$$\eta_m = C_1 e^{m\theta i} \tag{40}$$

$$\zeta_m = C_2 e^{m\theta i} \tag{41}$$

and these were substituted into equations (35) and (36). It is convenient to shift the index by one in (35) and (36).

$$\eta_{m+1} = a\eta_m + a\zeta_m + \alpha V_{m+1} \tag{42}$$

$$\zeta_{m+1} = -\beta\eta_m + c\zeta_m + \beta V_{m+1} \quad . \tag{43}$$

We also shift the index in (40) and (41) by one.

$$\eta_{m+1} = C_1 e^{(m+1)\theta i} \tag{44}$$

$$\zeta_{m+1} = C_2 e^{(m+1)\theta i} \tag{45}$$

Substituting (44) and (45) into (42) and (43) respectively yields

$$C_1 e^{\theta i} = aC_1 + aC_2 + \alpha e^{\theta i} \tag{46}$$

$$C_2 e^{\theta i} = -\beta C_1 + cC_2 + \beta e^{\theta i} \quad . \tag{47}$$

Now, (46) can be written

$$(e^{\theta i} - a)C_1 - aC_2 = \alpha e^{\theta i} \tag{48}$$

and (47) can be written

$$\beta C_1 + (e^{\theta i} - c)C_2 = \beta e^{\theta i} . \tag{49}$$

Equations (48) and (49) are two equations in two unknowns, C1 and C2. The solutions are

$$C_1 = (F + Gi)/(H + Si) \tag{50}$$

where

$$F = \alpha \cos 2\theta - c\alpha \cos\theta + a\beta \cos\theta \tag{51}$$

$$G = \alpha \sin 2\theta - c\alpha \sin\theta + a\beta \sin\theta \tag{52}$$

$$H = ac + a\beta + \cos 2\theta - (a + c)\cos\theta \tag{53}$$

$$S = \sin 2\theta - a\sin\theta - c\sin\theta \tag{54}$$

and

$$C_2 = (P + Qi)/(H + Si) \tag{55}$$

where

$$P = \beta \cos 2\theta - a\beta \cos\theta - \beta\alpha \cos\theta \tag{56}$$

$$Q = \beta \sin 2\theta - a\beta \sin\theta - \beta\alpha \sin\theta . \tag{57}$$

Then the amplitudes of the sin waves are

$$|C_1| = \sqrt{F^2+G^2/H^2+S^2} \tag{58}$$

$$|C_2| = \sqrt{P^2+Q^2/H^2+S^2} \quad . \tag{59}$$

2. <u>The Results</u>

The magnitudes of C1 and C2 were calculated on the computer for seven different periods, and the results are shown in TABLE VI.

These results show that when the errors in the signal are sinusoidal, the errors in the filtered values are also sinusoidal. The amplitudes of errors in the filtered values are much smaller than the measured values when the period is small. For large periods the amplitudes of the errors in the filtered values of y are approximately equal to the measured values of y. Periods of order eight (8) to sixteen (16) seconds appear to be the critical periods, where maximum errors in the filtered values of y and ydot occur.

As an example of the significance of a sinusoidal error in the measured value of ydot, suppose we have a sinusoidal error with an amplitude of ten (10) feet per second and a period of eight (8) seconds. This would result in an error in the filtered value of velocity of approximately four (4) feet per second, which would cause a miss distance of approximately 125 feet.

THETA	PERIOD (secs)	$\lvert C_1 \rvert$	$\lvert C_2 \rvert / \Delta t$
π	.25	.051	.024
$\pi/2$	.5	.073	.032
$\pi/4$	1	.134	.056
$\pi/8$	2	.265	.112
$\pi/16$	4	.537	.224
$\pi/32$	8	1.034	.392
$\pi/64$	16	1.239	.36
$\pi/128$	32	1.091	.192

Table VI

Calculated Amplitudes for Various Periods

C. EFFECTS OF TURNS

Two types of maneuvers are analyzed here.

First, during the final stage the plane can be expected to make a sequence of corrections, which we may expect to be small, sharp turns. Hence, we need to know the errors introduced in the velocity and position estimates by such maneuvers. As a typical turn we considered a plane flying at 350 feet per second turning at a rate of three (3) degrees per second for one second. This yields values

$$y(k) = 9.15/64 \; k^2 \quad ; k \le 8 \tag{60a}$$

$$\overset{o}{y}(k) = 18.3/8 \; k \quad ; k \le 8 \tag{60b}$$

$$y(k) = 9.15 + 18.3/8 \, (k-8) \quad ; k > 8 \tag{60c}$$

$$\overset{o}{y}(k) = 18.3 \quad ; k > 8 \tag{60d}$$

normal to the initial motion. The values along the initial course are not changed significantly.

The values of y and ydot and the filtered values of y and ydot were calculated, and the errors checked. The maximum errors were of the order .0005 feet for y and .0008 feet per second for ydot, which are negligible.

Second, we considered a plane in a constant turning movement. This is an unlikely maneuver for an aircraft to be performing, since the airplane flies on a straight line except for minor adjustments in the precision guidance

stage. For this reason higher errors were expected.

We calculated the errors in the filtered values of y and ydot, and the corresponding miss distances as a result of these maneuvers. It was found that errors in the filtered values of y caused a miss distance of approximately 100 feet, while errors in the filtered values of ydot caused a miss distance of approximately 1100 feet. This is a significant miss distance, but as stated earlier, it is not surprising since a constant turn by the plane should only occur in the preliminary stages.

V. CONCLUSIONS AND COMMENTS

Several conclusions and some comments are presented here as a means of summarizing this study.

This thesis had several purposes, focusing on the desire of the author to study mathematical methods and techniques that have been and/or will be useful when applied to a problem of interest to the Marine Corps. The techniques and methods used here are all known, but there is no place where the combination of numerical methods, ballistics, differential techniques, difference (filtering) equations, and statistics is available; they generally are scattered throughout various fields.

A. CONCLUSICNS

Most of the results obtained in this thesis were, in a sense, expected. The range of validity of linear approximations to changes was surprisingly large, however.

In our very limited checking, the fraction eight-tenths of wind at altitude, actually given to us as part of the "lore" was found to be near the best choice for use in the absence of stratified wind data. Its validity, however, can be checked only by some actual testing and studying of actual wind structures.

We also found that errors in various initial conditions result in errors in the impact point of the bomb which are

linear over a fairly wide range. It does not seem to make much difference which technique we use to estimate these. Under the conditions of our trajectories an error or change of one foot per second in horizontal velocity led to an error or change in point of impact of about 32 feet, roughly independent of the direction of the error. A value of zdot of one (1) foot per second at release led to an increase in range of about 2.26 feet. The miss distances were found to be nearly linear over a wide range of values for the initial errors.

The analysis of the difference equations was used to show how a random error in measurement leads directly to a random error in the point of impact. From the standard deviation of the radar measurements we derived the standard deviation of the estimates for y and ydot and their correlation. With these and the sensitivity coefficients we obtained the standard deviation of the associated misses. Under the conditions we found that if the radar has a random error in measurement with standard deviation σ_v, then there will be a resulting error in point of fall with standard deviation $\sigma_{y_f} = 3\sigma_v$. For example, if the radar noise has an rms (root-mean-square) value of ten (10) feet, the point of impact will have an associated rms value of thirty (30) feet. Most of this results from the error in predicting ydot, though y and ydot have a fairly large positive correlation, and hence the error is larger than if they were independent.

Also we found that the presence of a sinusoidal error in the measurements resulted in errors which are sin waves of varying amplitude in the position and velocity estimates. We found that a sinusoidal error with an amplitude of ten (10) feet and a period of eight (8) seconds will cause a periodic error in the point of fall with a maximum amplitude of 125 feet and an rms value of about 96 feet. The actual

error depends on the particular time of release. Sinusoidal errors with periods of one (1) second or less have little effect.

The effects of a small, sharp turn by the aircraft on the position and velocity estimates were found to be fairly negligible. On the other hand, the errors in position and velocity estimates for an aircraft in a constant turn were found to be so large that they would generally preclude the use of the present filtering equations (For a turn rate of three (3) degrees per second a miss distance of 1100 feet would result, principally from the error in the filtered value of ydot.); however, this type of maneuver should never occur during the final precision guidance stage.

B. COMMENTS

A number of comments pertaining to our study should be made. First, a number of errors and their sources were not studied, primarily since little could be done at this level from a mathematical or a scientific nature to check or improve those errors. Such errors include radar alignment error, the actual bomb alignment on the aircraft, the dampness or dryness of the bomb itself, and the behavior of air currents in the atmosphere.

Proper radar alignment depends upon proper training and discipline of the personnel involved in setting up the equipment, as well as good surveys prior to setting up the radar. These are factors that must be considered in the field where the equipment is operated.

The assumption in this thesis that the tail vanes kept the bomb aligned with its axis tangential to the velocity

vector relative to the air is felt to be probably the greatest source of random error in the system. The bomb comes out with angular motion and may even tumble momentarily; while this occurs at the time of low speed and thinnest air the velocity changes at this time cause large integrated effects. Any analysis of this attempting to model the aerodynamic forces and moments due to various initial configurations would be of doubtful value. The only feasible way seems to be numerical experimentation with the actual configuration, combined with dropping the bombs and observing the paths.

We also limited ourselves to one set of standard conditions; there is little point to carrying out more computations of the same type unless the computations are associated with an actual system. Then many of these should be carried out over the range of parameters expected to occur, such as altitude of plane and of target; speed of the plane, and particularly with the best approximation for the drag coefficient, as a function of Mach number.

It must be noted that this research took a general look at a sophisticated problem, and in doing so touched only the surface in terms of investigating error sources. A system as expensive and as sophisticated as the AN/TPQ-27 requires intensive research and testing, as it has received and will continue to receive.

Finally, an important facet of this work was to provide a familiarization, or an awareness document for ready reference to those individuals who need some introduction to the mathematics used to analyze the system. In that sense the intended goal of the research was achieved.

LIST OF REFERENCES

1. Department of The Navy, Naval Electronics Systems Command, Contract #N00039-75-C-0460, AN/TPQ-27 Radar Course Directing Central Computer Program Performance Specification, 24 December 1975.

2. Gerald, Applied Numerical Analysis, p. 115 to 119, Addison-Wesley Publishing Company, 1973.

3. Lentz, Robert Eugene, Improvement of AN/TPQ-27 Filter And Control Techniques, Electrical Engineering Thesis, Naval Postgraduate School, Monterey, California, 1974.

4. RCA Government Systems Division, Moorestown, New Jersey, '27' PM-2000-89, '27' Program Memorandum, Some Ballistic Wind To Wind-At-Altitude Ratios Calculated From Camp Pendleton Stratified Wind Data, by J. Liston, 18 June 1976.

5. Leabo, Basic Statistics, p. 95 and 96, Richard D. Irwin, Inc., 1975.

6. O'Leary, Ronald Neil, and Petersen, Kenneth Burton, An Analysis of Hit Probabilities of a TPQ System And a Comparison of The TPQ-10 And TPQ-27 Systems, M.S. Thesis, Naval Postgraduate School, Monterey, California, 1973.

7. Bliss, Mathematics for Exterior Ballistics, p. 63 to 97, John Wiley And Sons, Inc., 1944.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, California 93940	2
3.	Department Chairman, Code 53 Department of Mathematics Naval Postgraduate School Monterey, California 93940	1
4.	Professor Frank D. Faulkner, Code 53 Fa Department of Mathematics Naval Postgraduate School Monterey, California 93940	9
5.	Captain Ronald Chambless 411 Canal Street Paragould, Arkansas 72450	1

Thesis
C3457 Chambless 174740
c.1
A numerical analysis
of errors in a bombing
system.